In this lesson, you will learn about dividing fractions. Take a look at the model shown:

You can use this model to divide certain fractions. For example, to determine $\frac{3}{7} \div \frac{3}{5}$, count the number of sections in the top bar from left to right until you reach the right edge of $\frac{3}{7}$. There are 15 equal sections. Now count the number of sections in the bottom bar from left to right until you reach the right edge of $\frac{3}{5}$. There are 21 equal sections. This means that your answer is $\frac{15}{21}$, or $\frac{5}{7}$.

How does this work? Maybe when you have completed this lesson you can figure it out.
Problem 1  Division: Please Explain Yourself!

Division of whole numbers means to ask how many groups of a certain size are contained in a number.

The expression $12 \div 3$ means you are trying to determine how many groups of 3 are in 12. A physical model and number line model are shown.

**Physical Model**

1 group of 12

4 groups of 3

**Number Line Model**

1 group of 12

4 groups of 3

$12 \div 3 = 4$

There are 4 groups of 3 in 12.
When you divide with fractions, you are asking the same question.

The expression \( 2 \div \frac{1}{2} \) is asking how many halves are in 2.

**Physical Model**

\[ \frac{1}{2} \quad \frac{1}{2} \]

**Number Line Model**

There are four \( \frac{1}{2} \) parts in 2, so \( 2 \div \frac{1}{2} = 4 \).

The expression \( \frac{1}{2} \div 2 \) is asking how many groups of 2 are in \( \frac{1}{2} \).

**Physical Model**

\[ \frac{1}{2} \]

**Number Line Model**

There is \( \frac{1}{4} \) of a group of 2 in \( \frac{1}{2} \), so \( \frac{1}{2} \div 2 = \frac{1}{4} \).
1. Write a sentence to describe what the division expression is asking. Then, draw a diagram to represent the division problem. Finally, calculate the quotient, and write a sentence to describe your answer. Use your fraction strips to help you draw the model.

   a. \( \frac{3}{4} \div \frac{1}{4} \)

   b. \( \frac{3}{2} \div \frac{1}{4} \)

   c. \( 3 \div \frac{3}{4} \)
2. How can you check each of your answers in Question 1 to make sure you were correct? Explain your reasoning.
3. Jamilla is throwing a small party. She has 4 pizzas and decides that everyone at her party should receive a serving size that is $\frac{3}{5}$ of a pizza. Jamilla says she has $6 \frac{2}{3}$ servings, but Devon says she has $6 \frac{2}{5}$ servings. Draw a diagram of the situation, and solve for the quotient to determine who is correct. Then explain why one person is not correct.

4. Calculate each product. Show your work.
   
   a. $\frac{1}{3} \times \frac{3}{1}$
   b. $\frac{2}{3} \times \frac{3}{2}$

   c. $13 \times \frac{1}{13}$
   d. $\frac{7}{8} \times \frac{8}{7}$

   e. $\frac{1}{5} \times \frac{5}{1}$
   f. $\frac{6}{9} \times \frac{9}{6}$

5. What do you notice about each product in Question 4?
When you reverse the numbers in the numerator and denominator of a fraction, you form a new fraction called the *reciprocal*. The *reciprocal* of a number is also known as the *multiplicative inverse* of the number. The *multiplicative inverse* of a number \( \frac{a}{b} \) is the number \( \frac{b}{a} \), where \( a \) and \( b \) are nonzero numbers. The product of any nonzero number and its multiplicative inverse is 1. The fractions you multiplied in Question 4 are reciprocals of each other.

The **Multiplicative Inverse Property** states: \( \frac{a}{b} \times \frac{b}{a} = 1 \), where \( a \) and \( b \) are nonzero numbers.

6. Which number is its own reciprocal?

7. Which number has no reciprocal? Explain your reasoning.

8. Alexa wrote the reciprocal of the mixed number incorrectly. Explain why she is incorrect and provide the correct reciprocal.

Alexa
Given \( \frac{3}{5} \)
The reciprocal is \( \frac{5}{3} \).
Karen said, “I wish everything could be as easy as dividing by 1.” She tried her “dividing by 1” method on the division of fraction problem.

\[
\frac{5}{8} \div \frac{3}{4}
\]

“If I can turn the divisor of \(\frac{3}{4}\) into one, then the problem can be solved. I can multiply both fractions by the reciprocal of \(\frac{3}{4}\), which is \(\frac{4}{3}\), to create 1.”

9. Analyze Karen’s method for dividing fractions. Describe the steps in the dashed circles.

\[
\begin{align*}
\frac{5}{8} \div \frac{3}{4} &= \frac{5}{8} \times \frac{4}{3} \\
&= \frac{5 \times 4}{8 \times 3} \\
&= \frac{20}{24} \\
&= \frac{5}{6}
\end{align*}
\]

10. Write a rule based on Karen’s method that you can use to calculate the quotient in the division of fraction problem.

11. Calculate each quotient. Show your work. Make sure your answer is in simplest form and make certain none of your answers are improper fractions.

a. \(\frac{3}{4} \div \frac{1}{3}\)

b. \(\frac{3}{8} \div \frac{1}{4}\)

c. \(\frac{5}{6} \div \frac{2}{3}\)

d. \(\frac{7}{8} \div \frac{3}{4}\)

e. \(\frac{11}{12} \div \frac{2}{3}\)

f. \(\frac{9}{10} \div \frac{3}{5}\)
12. Draw a diagram for each problem. Then, match the expression with each problem. Finally, solve the problem.

<table>
<thead>
<tr>
<th>Number Sentences</th>
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<tbody>
<tr>
<td>( \frac{5}{8} \div 2 )</td>
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a. Georgianne has \( \frac{5}{8} \) cups of raisins. Each batch of trail mix requires \( \frac{1}{4} \) cup of raisins. How many batches of trail mix can she make?

b. Georgianne has \( \frac{5}{8} \) of a batch of trail mix. She plans to share the trail mix equally with her friend Jackie. How much of the batch will each receive?

c. Georgianne has to travel \( \frac{5}{8} \) of a mile to the store. She can walk \( \frac{1}{4} \) of a mile in an hour. How many hours will it take her to travel to the store?
d. Georgianne has to travel $\frac{5}{8}$ of a mile. She is going to bike halfway and skate halfway. How far will she bike?

13. Draw a diagram for each problem situation. Then, write the appropriate number sentence.

a. How many students can be served with 4 cups of trail mix if each student gets $\frac{1}{2}$ of a cup of trail mix?

b. How many $\frac{1}{4}$-cup servings of trail mix can you make with 4 cups?

c. How many $\frac{1}{3}$-cup trail mix servings can you make with 4 cups?

d. Do you notice a pattern? Explain your reasoning.
14. You have 4 cups of trail mix. If each student receives:
   a. \( \frac{2}{3} \) cup, how many students are there?
   b. \( \frac{2}{5} \) cup, how many students are there?
   c. \( \frac{4}{5} \) cup, how many students are there?
   d. \( \frac{4}{7} \) cup, how many students are there?

15. What pattern do you notice? Explain your reasoning.

16. How many students are there if:
   a. you have \( \frac{1}{2} \) cup of trail mix and each student gets \( \frac{1}{4} \) cup?
   b. you have \( \frac{1}{3} \) cup of trail mix and each student gets \( \frac{1}{6} \) cup?
   c. you have \( \frac{2}{3} \) cup of trail mix and each student gets \( \frac{1}{6} \) cup?
   d. you have \( \frac{3}{5} \) cup of trail mix and each student gets \( \frac{1}{10} \) cup?
   e. you have \( \frac{2}{3} \) cup of trail mix and each student gets \( \frac{2}{9} \) cup?
   f. you have \( \frac{2}{3} \) cup of trail mix and each student gets \( \frac{3}{9} \) cup?

**Talk the Talk**

1. How is the quotient of $12 \div \frac{1}{3}$ related to the quotient of $12 \div \frac{2}{3}$? Explain your reasoning.

2. Determine the quotient for each. Then, describe any patterns that you notice.

   \[
   \begin{align*}
   6 \div \frac{1}{2} & \quad 6 \div \frac{1}{4} & \quad 6 \div \frac{1}{8} & \quad 6 \div \frac{1}{16}
   \end{align*}
   \]

3. Lindsey asks her group to test an idea she has about the division of fractions. She thinks that each division problem shown has the same answer.

   \[
   \begin{align*}
   8 \div 6 \\
   8 \div 6 \\
   9 \div 9 \\
   8 \div 6 \\
   5 \div 5 \\
   8 \div 6 \\
   11 \div 11 \\
   8 \div 6 \\
   3 \div 3
   \end{align*}
   \]

   Is Lindsey correct? Explain why or why not.

Be prepared to share your solutions and methods.